Graduate Analysis I

Mid-term Examination

Semester 1, 2008-2009

Department of Mathematics,
National University of Singapore

Time allowed: $1\frac{1}{2}$ hours

Question 1. [20 marks]

Find the Lebesgue measure of the set $E = \overline{\{\sin n : n \in \mathbb{Z}\}}$ where $\mathbb{Z}$ is the set of all integers.

Question 2. [20 marks]

Let E be a measurable set in $\mathbb{R}$ and $0 < |E| = q < \infty$. Show that, for any $0 < q < p$, there exists a perfect subset $F \subset E$ such that $|F| = q$.

Question 3. [20 marks]

Find the bounded variation of the function f over $[0, 2]$

$$f(x) = \begin{cases} x - 1, & \text{if } x < 1, \\ 10, & \text{if } x = 1, \\ x^2, & \text{if } x > 1. \end{cases}$$

Question 4. [20 marks]

Given an example to show that from the fact that $f^2(x)$ is measurable, one cannot conclude that f is measurable. But do show that if f^3 is measurable on a measurable set E, then f is also measurable on E.

Question 5. [20 marks]

Let E be a measurable set in $\mathbb{R}^n$ with $|E| < \infty$. Suppose f_n, g_n are two sequences of almost everywhere finite measurable functions such that $f_n \to f$ and $g_n \to g$ in measure as $n \to \infty$. Show that $f_n g_n \to fg$ in measure as $n \to \infty$.